Approximate Inference

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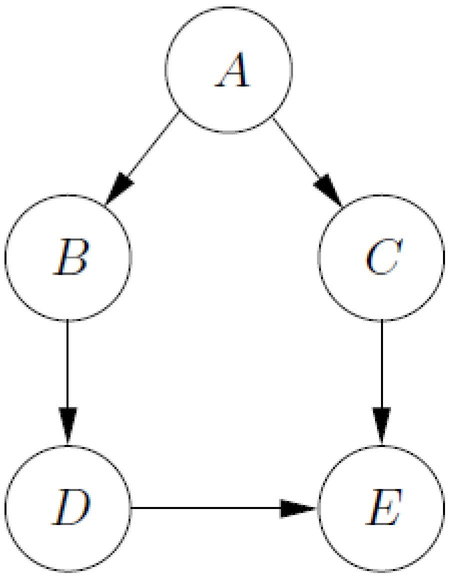
Probabilistic Logic Sampling



B \ A	y	n
y	0.3	0.8
n	0.7	0.2

$P(B \mid A)$

C \ A	y	n
y	0.7	0.4
n	0.3	0.6

$P(C \mid A)$

D \ B	y	n
y	0.5	0.1
n	0.5	0.9

$P(D \mid B)$

D \ C	y	n
y	(0.9, 0.1)	(0.999, 0.001)
n	(0.999, 0.001)	(0.999, 0.001)

$P(E \mid C, D)$

Probabilistic Logic Sampling

	CDE							
AB	yyy	yyn	yny	ynn	nyy	nyn	nny	nnn
yy	4	0	5	0	1	0	2	0
yn	2	0	16	0	1	0	8	0
ny	9	1	10	0	14	0	16	0
nn	0	0	4	0	0	0	7	0

$$P(E) \approx \left(\frac{N(E=y)}{N}, \frac{N(E=n)}{N}\right) = \left(\frac{99}{100}, \frac{1}{100}\right) = (0.99, 0.01).$$

Probabilistic Logic Sampling

1. Let $(X_1, \ldots, X_n)$ be a topological ordering of the variables.
2. For $j = 1$ to N:
   a) For $i = 1$ to n:
      - Sample a state x_i for X_i using $P(X_i \mid \text{pa}(X_i) = \pi)$, where π is the configuration already sampled for $\text{pa}(X_i)$.

   b) If $\mathbf{x} = (x_1, \ldots, x_n)$ is consistent with $\mathbf{e}$, then

$$N(X_k = x_k) := N(X_k = x_k) + 1,$$

   where x_k is the state that was sampled for X_k.
3. Return:

$$P(X_k = x_k \mid \mathbf{e}) \approx \frac{N(X_k = x_k)}{\sum_{x \in \text{sp}(X_k)} N(X_k = x)}.$$

Likelihood Weighting

$$P(\mathcal{U}, \mathbf{e}) = \underbrace{\prod_{X \in \mathcal{U} \setminus \mathcal{E}} P(X \mid \mathrm{pa}(X)', \mathrm{pa}(X)'' = \mathbf{e})}_{\text{Part 1}}$$

$$\times \underbrace{\prod_{X \in \mathcal{E}} P(X = e \mid \mathrm{pa}(X)', \mathrm{pa}(X)'' = \mathbf{e})}\,.$$

$$\text{Sampling distribution} = \prod_{X \in \mathcal{U} \setminus \mathcal{E}} P(X \mid \mathrm{pa}(X)', \mathrm{pa}(X)'' = \mathbf{e}),$$

$$w(\mathbf{x}, \mathbf{e}) = \prod_{E \in \mathcal{E}} P(E = e \mid \mathrm{pa}(X) = \pi),$$

where π is the configuration of $\mathrm{pa}(X)$ specified by $\mathbf{x}$ and $\mathbf{e}$.

Likelihood Weighting

1. Let $(X_1, \ldots, X_n)$ be a topological ordering of the variables.
2. For $j = 1$ to N:
   a) w:=1.
   b) For $i = 1$ to n:
   - Let $\mathbf{x}'$ be the configuration of $(X_1, \ldots, X_{i-1})$ specified by $\mathbf{e}$ and the previous samples.
   - If $X_i \notin \mathcal{E}$, then:
     - Sample a state x_i for X_i using $P(X_i \mid \text{pa}(X_i) = \pi)$, where $\text{pa}(X_i) = \pi$ is consistent with $\mathbf{x}'$.
   - else
     $w := w \cdot P(X_i = e_i \mid \text{pa}(X_i) = \pi)$, where $\text{pa}(X_i) = \pi$ is consistent with $\mathbf{x}'$.

   c) $N(X_k = x_k) := N(X_k = x_k) + w$, where x_k is the sampled state for X_k.
3. Return:

$$P(X_k = x_k \mid \mathbf{e}) \approx \frac{N(X_k = x_k)}{\sum_{x \in \text{sp}(X_k)} N(X_k = x)}.$$